# **Inductive Measurement of Plasma Jet Electrical Conductivity**

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An inductive probing scheme, originally developed for shock tube studies, has been adapted to measure explosive plasma jet conductivities. In this method, the perturbation of an applied magnetic field by a plasma jet induces a voltage in a search coil, which, in turn, can be used to infer electrical conductivity through the inversion of a Fredholm integral equation of the first kind. A 1-in. (25.4-mm)-diam probe was designed and constructed, and calibration was accomplished by firing an aluminum slug through the probe using a light-gas gun. Exploratory laboratory experiments were carried out using plasma jets expelled from 15-g high-explosive shaped charges. Measured conductivities were in the range of 3 kS/m for unseeded octol charges and 20 kS/m for seeded octol charges containing 2% potassium carbonate by mass.

## Nomenclature

 $D_m$  = magnetic diffusivity  $f(\xi)$  = distribution function g(s) = measured signal

 $I, I_c$  = field coil current during test and

during calibration

 $I_{\rm sp}$  = specific impulse

 $K(s, \xi)$  = kernel L = length

 $L_0$  = characteristic length

l = length of conductive region behind the shock

N = sample size R = resistance  $R_d$  = damping resistance  $Re_m$  = magnetic Reynolds number

s = distance from search coil to the shock

t = time

U = velocity of the shock

 $\begin{array}{lll} u & = & \text{particle velocity behind the shock} \\ u_c & = & \text{velocity of the calibration slug} \\ u_0 & = & \text{characteristic plasma velocity} \\ V_c & = & \text{calibration response function or kernel} \\ V_{cp} & = & \text{peak value of calibration slug response} \\ V_p & = & \text{peak value of the explosive test voltage curve} \end{array}$ 

V(s) = output signal of the search coil

x = distance from the search coil to an arbitrary point

 $\begin{array}{ccc} & & \text{behind the shock} \\ \delta & = & \text{differential change} \\ \varepsilon & = & \text{finite measurement error} \end{array}$ 

 $\mu_0$  = magnetic permeability of free space  $\xi$  = distance from an arbitrary location x to

the shock, s - x

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 $\rho$  = resistivity

 $\sigma$  = electrical conductivity

 $\sigma_c$  = calibration slug conductivity

 $\sigma_0$  = characteristic plasma electrical conductivity

 $\sigma^*$  = maximum conductivity

 $\Phi$  = pulse area of measured voltage curve

 $\Phi_c$  = pulse area of calibration slug response function

 $\Phi(s)$  = integrated-induced voltage signal

 $\Psi_1, \Psi_2$  = function dependent on the conductivity distribution

Subscripts

c = calibration parameter

0 = appropriate characteristic value for the problem

## Introduction

EASUREMENT of plasma jet electrical conductivity has utility in the development of explosively driven magnetohydrodynamic (MHD) energy converters as well as magnetic flux compression reaction chambers for nuclear/chemical pulse propulsion and power. Reactors of this type can accomplish two important functions: 1) collimation and reflection of the hot diamagnetic plasma for direct thrust production with minimal ablative effects and 2) electric power generation by direct energy conversion.<sup>1</sup> Of crucial importance to the complex underlying MHD processes in field compression reactors is the magnetic Reynolds number  $Re_m$ , the value of which depends on the product of plasma electrical conductivity and velocity,  $Re_m = \mu_0 \sigma_0 u_0 L_0 = u_0 L_0 / D_m$ . Because the magnetic Reynolds number is inversely related to the magnetic diffusivity  $D_m$ , it is desirable to obtain high electrical conductivity and high expansion velocity to achieve low flux diffusion losses for optimum reactor performance. Indeed, flux diffusion losses into the plasma will be severe unless  $Re_m \gg 1$ . As such, a thorough understanding of MHD phenomena at high magnetic Reynolds number is essential to the design of flux compression reactors as well as explosively driven MHD converters.

Explosive sources offer a simple and convenient method for producing high-speed highly conductive plasma jets. Available empirical data for the electrical conductivity of explosive plasma jets are limited, but there is evidence to indicate that sufficiently high magnetic Reynolds numbers can be produced. For instance, experiments on 44-deg cavity charges of composition-4 by Burnham and Marshall, Burnham, and Burnham and Marshall have revealed that a cohesive plasma slug is ejected at speeds on the order of 30 km/s with a core electrical conductivity of 10 kS/m. Independent explosive-driven MHD experiments by Jones and McKinnon indicated the production of magnetic Reynolds number approaching

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unity in moderate scale devices.<sup>5</sup> Explosive plasma sources developed by Baum have produced plasma flow velocities of 30 km/s and conductivities of 30 kS/m, which in a laboratory scale 25-mm-diam tube yield a magnetic Reynolds number of 28 (see Ref. 6). The explosive compressor developed by Voitenko produces comparable shock velocities and is likely to have a comparable range of  $Re_m$  (Ref. 7). Furthermore, a concept for explosive-driven collapse of a lined cylindrical cavity, as proposed by Kosi et al.,<sup>8</sup> Wild,<sup>9</sup> and Dunne et al.<sup>10</sup> has shown considerable promise as well.

Based on this evidence, research had proceeded with the development of a simple and reliable inductive probe for measuring the electrical conductivity of explosive plasma jets. Various techniques have been used in the past to measure directly the conductivity of ionized gases. 11-18 Some methods employ the principle that a magnetic field is distorted by a moving plasma, 11,12 whereas others utilize the principle that the resistance of the electrically conducting gas dissipates energy from an oscillating magnetic field. 13,14 A more direct approach using biased contact electrodes is possible, 14-18 but it is well known that this methodology suffers from large surface resistances, <sup>11</sup> and an electrodeless technique is preferred. Therefore, it was decided to adapt the inductive probing technique developed by Lin et al. for shock tube studies. 11 In this method, the perturbation of an applied magnetic field by a plasma jet induces a voltage in a search coil, which, in turn, can be used to infer electrical conductivity through the inversion of a Fredholm integral equation of the first kind. This paper describes the design, development, and calibration of a 25.4-mm-diam probe and the results of exploratory experiments using explosive plasma jets expelled from 15-g shaped charges formed from seeded and unseeded octol.

## **Theory of Inductive Probe**

#### **Fundamental Principles of Operation**

The inductive probe (Fig. 1) is a simple electromagnetic system consisting of two adjacent coils wound on a nonmagnetic tube. The field coil, that is, solenoid, is first excited by a dc power supply to introduce an axisymmetric magnetic field inside the tube. The resulting field lines are shown in Fig. 1. A small search coil is situated slightly upstream of the field coil to pick up the electromagnetic disturbance produced by passage of the plasma jet.

As the plasma jet enters the field region, it displaces the magnetic field lines due to its diamagnetic properties. That is, eddy currents are established in the plasma, which resist magnetic field penetration. This perturbation, or compression, of the magnetic field lines induces an emf in the search coil. The magnitude of the emf is representative of the degree of field displacement and, given the jet velocity, can be correlated with the plasma electrical conductivity. Actual recovery of the electrical conductivity distribution from the induced emf signal requires the difficult inversion of a Fredholm integral equation of the first kind, as shown in the following subsection devoted to the mathematical theory of probe operation.

## **Mathematical Theory of Operation**

Following the mathematical analysis of Lin et al.<sup>11</sup> for the inductive probe, one arrives at a Fredholm integral equation of the first

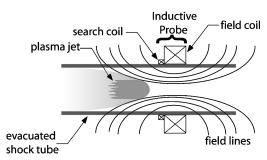


Fig. 1 Inductive probe principle of operation.

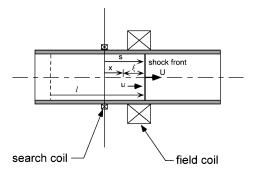


Fig. 2 Parameters relevant to analysis of inductive probe operation.

kind for the conductivity  $\sigma$ ,

$$\Phi(s) = \frac{UuI}{u_c^2 I_c \sigma_c} \int_0^l V_c(s - \xi) \sigma(\xi) \,\mathrm{d}\xi \tag{1}$$

where  $\Phi(s)$  is the integrated induced voltage signal on the search coil

$$\Phi(s) = \int_0^s V(s) \, \mathrm{d}s \tag{2}$$

and V(s) is the output signal of the search coil. The parameters relevant to the mathematical analysis are shown in Fig. 2. The major parameters are as follows: the length of the conductive region behind the shock I; the distance from the search coil to the shock s; the distance from the search coil to an arbitrary point behind the shock x; the distance from an arbitrary location x to the shock,  $\xi = s - x$ ; the velocity of the shock U; the particle velocity behind the shock u; the field current during a test I; the velocity of the calibration slug  $u_c$ ; the field current during calibration  $I_c$ ; the electrical conductivity of the calibration slug  $\sigma_c$ ; the calibration slug response function or kernel  $V_c$ ; and the unknown plasma electrical conductivity distribution  $\sigma$ 

The Fredholm integral equation of the first kind has the general form

$$g(s) = \int_{a}^{b} K(s, \xi) f(\xi) \,\mathrm{d}\xi \tag{3}$$

where g(s) is a known function, that is, the measured signal, and  $f(\xi)$  is the unknown distribution function. In general, closed-form analytical inversions can be obtained only for special functions, and in all other cases, one must resort to numerical methods.

In some cases, interpretation of the experimental data can be facilitated by an approximate treatment of the integral equation. For example, Lin et al. showed that it is possible to express the ratio between the maximum conductivity  $\sigma^*$  behind the shock wave and the calibration slug conductivity  $\sigma_c$  as

$$\frac{\sigma^*}{\sigma_C} = \left(\frac{u_c^2 I_c V_p}{U u I V_{cp}}\right) \Psi_1 \tag{4}$$

or

$$\frac{\sigma^*}{\sigma_C} = \left(\frac{u_c^2 I_c \Phi_p}{U u I \Phi_{cp}}\right) \Psi_2 \tag{5}$$

where  $V_{cp}$  and  $\Phi_c$  are the peak and the pulse area of the calibration slug response function, respectively;  $V_p$  and  $\Phi$  are the peak and the pulse area of the calculated voltage curve; and  $\Psi_1$  and  $\Psi_2$  are functions dependent on the conductivity distribution. Various results for different conductivity distributions are tabulated in the literature.  $^{11}$ 

For the special case where the conductivity distribution is a step function and the probe response function is Gaussian, both  $\Psi_1$  and  $\Psi_2$  are unity. When the conductivity distribution deviates from a step function, both  $\Psi_1$  and  $\Psi_2$  can become complicated functions

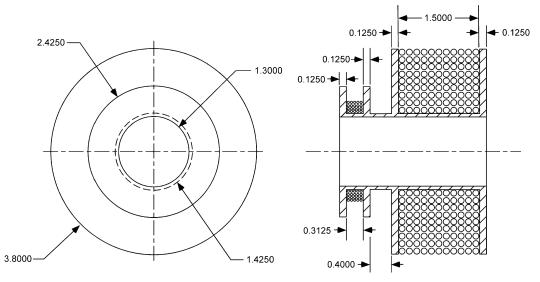


Fig. 3 Details of inductive probe, nylon material, measurements in inches.

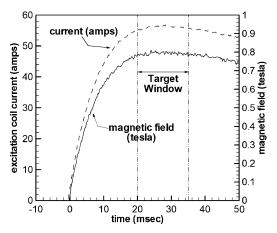


Fig. 4 Measured coil current and field intensity waveforms.

that may only be evaluated numerically. Equation (5) is thought of as a more conservative calculation because the value of  $\Psi_2$  is unity for a wider range of conductivity distributions. <sup>11</sup> At any rate, note that, when the variation of conductivity is not too great,  $\Psi_2$  will be unity plus a small correction term. This behavior turns out to be quite fortuitous for the purpose of analyzing the plasma jets encountered in our experiments.

# **Probe Development**

## Design

The detailed design of the inductive probe is shown in Fig. 3. The basic structure is a nylon coil-form, which has been machined to accept two coil windings. The coil-form has a bore diameter of 33 mm and a wall thickness of 3.2 mm. A 900-turn magnet field coil was wound from number 18 enameled magnet wire. The resulting solenoid has a length of 38 mm and an outer diameter of 96.5 mm. The search coil consists of dual 25-turn windings of number 28 enameled magnet wire. These windings can be connected in series or in a push–pull configuration as will be discussed.

The excitation field coil was designed to produce a solenoid magnetic field between 0.5 and 1 T inside the bore. A 20-kW dc power supply was used to drive the excitation coil at 300 V (maximum), and the operating time was limited to 1 s to minimize resistive heating.

The field coil was calibrated by measuring the time varying current when activated by a solenoid switch. The time varying magnetic field in the bore was also measured using a Hall probe connected

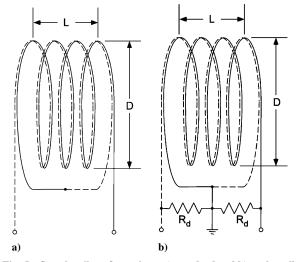


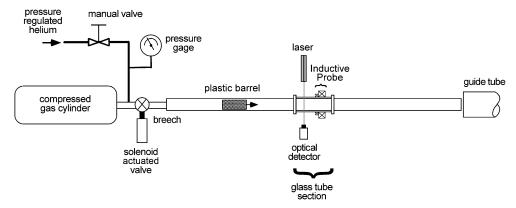
Fig. 5 Search coil configurations: a) standard and b) push-pull.

to a Gauss meter. The resulting waveforms are shown in Fig. 4. A rise time of 20 ms, a peak current of 56 A, and a peak field intensity of 0.8 T were observed. Reproducibility of these waveforms was excellent. The waveforms remain relatively flat around the peak values for about 15 ms, after which they gradually decay and stabilize to about 63% of their maximum values. Therefore, a 15-ms target window was available in which to probe the plasma jet at maximum field strength.

In the shock tube experiments of Lin et al., electrostatic effects were encountered that were associated with nonuniform charge distributions in the ionized gas.  $^{11}$  This effect, although of great physical interest, introduces a finite capacitance between the search coil and the plasma and could lead to spurious signals. This difficulty was overcome by connecting dual-turn windings in a push–pull configuration, as shown in Fig. 5. The center tap is connected to ground, and  $1\text{-}k\Omega$  shunt resistors are used to achieve critical damping. In this configuration, the capacitive pickup from the two ends of the coil is canceled, whereas the inductive pickup is unaffected. A similar strategy was implemented in the probe design so that either a 50-turn standard connection or a dual 25-turn push–pull connection could be utilized, as needed.

## Calibration

To calibrate the inductive probe, a metal slug of known electrical conductivity was fired through the probe. The velocity of



Experimental apparatus schematic used for probe calibration.

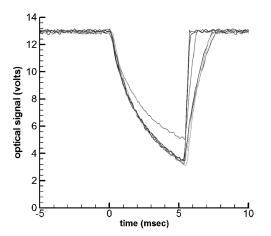


Fig. 7 Time-of-flight waveforms from calibration experiments.

ration. this calibration data will be used in future tests where one additional reading is taken, the random uncertainty range about this additional reading that the parent population mean will fall, with 95% confidence, is  $2150 \pm 401$  cm/s (Ref. 19).

the slug must also be measured. To satisfy this requirement, a simple compressed gas gun using 1034-kPa helium stored in a small cylinder was developed. The helium was released into the gun breech by a solenoid-actuated valve, which could be triggered electronically, and a 102-mm-long, 25.4-mm-diam aluminum cylinder was propelled down a plastic barrel and through a glass tube holding the probe assembly. The resistivity of the aluminum was  $\rho = 3 \times 10^{-8} \,\Omega \cdot \text{m}$ . The velocity of the slug was inferred from the obstruction of a laser beam. A digital computer controlled firing sequence was implemented incorporating the appropriate timing delays, and the various signal waveforms were captured on a digital oscilloscope. A schematic of the experimental apparatus is shown

The timing for the calibration experiment was crucial. For example, to hit the target window it was necessary to trigger accurately the excitation field coil 20 ms before the slug entered the active probe region; otherwise, the probe would not be at maximum field strength at the appropriate moment. This timing problem was summarily resolved after several test shots.

To measure the slug velocity, a laser beam was directed through the glass tube and onto a fast-response optical detector. When the slug entered the glass tube, the laser beam was obstructed and the detector signal fell. After slug passage, the beam path was cleared and the detector signal returned to its normal value. Therefore, the time of flight of the slug as it passed the beam could be determined, and the velocity inferred from the known slug length. The captured detector waveforms are shown in Fig. 7 from which an average time of flight of 4.75 ms was observed. The velocity of the slug ranged from 1900 to 2400 cm/s. The mean value of the seven tests was 2150 cm/s, with a standard deviation of 164 cm/s. When only random uncertainty effects are taken into account, the interval within which one would expect, with 95% confidence, the mean value of the parent population to fall is  $2150 \pm 152$  cm/s. However, because

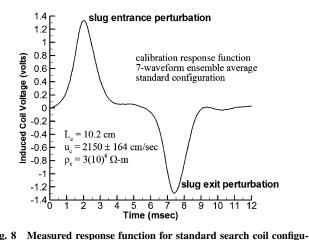


Fig. 8

The measured response function of the standard configuration search coil (50 turns) is shown in Fig. 8. This waveform, which represents an ensemble average of seven waveforms, consists of two perturbations. The first is the entrance perturbation associated with initial field displacement as the probe enters the active probe region. The second is the exit perturbation associated with restoration of the original field as the slug leaves the active probe region. When the slug fills the active region completely, there is no signal because the displaced field is quasi steady. Only the entrance portion of the response function is needed for inversion purposes.

The response function is very nearly Gaussian, as can be seen from the fit shown in Fig. 9. Here, transformation to spatial coordinates is made using the slug velocity. The response function is observed to reach a peak value of about 1.3 V when the leading edge of the slug is about 3.5 cm into the active coil region. The standard deviation of the Gaussian fit is 1.8 cm, indicating that the probe has an effective resolving power of approximately twice that value, or 4-cm. The leading edge of the plasma jet is expected to be similar to a step function, with the maximum value of the plasma jet conductivity falling within this 4-cm range; therefore, this resolving range is more than sufficient to measure the leading portion of a plasma jet.

The measured response functions for the push-pull search coil configuration (25-turn segments) are shown in Fig. 10. Again, these waveforms represent an ensemble average of seven independent tests. In this case, a response signal was obtained from each segment of the push-pull coil circuit, where the output from each segment is referenced to the grounded center tap. Therefore, the entrance

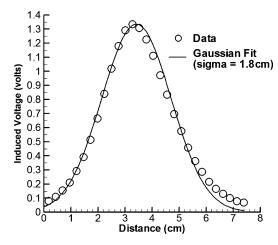


Fig. 9 Gaussian fit of response function for standard search coil configuration.

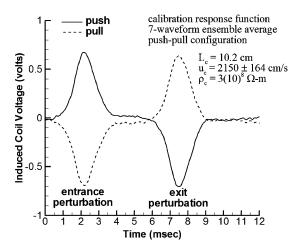


Fig. 10 Measured response function for push-pull coil configuration.

perturbation is positive for the push segment and negative for the pull segment. The exit perturbations are also of opposite sign.

The response functions are again found to be nearly Gaussian, and the peak signal values are exactly half of that obtained for the standard configuration, as expected. The effective resolving power is also identical to the standard configuration.

## **Plasma Jet Experiments**

With probe development completed, exploratory laboratory experiments on explosive plasma jets were initiated. In these experiments, 15-g shaped charges (included cone angle of 44 deg) composed of octal [75% Her Majesty's explosive (HMX) (ocahydrotetranitro-tetrazacine)/25% trinitrotoluene (TNT)] with various amounts of seed material (0, 1, and 2% potassium carbonate by mass) were investigated. All charges were prepared by Accurate Arms Company of McEwen, Tennessee. Based on previous experiments with similar explosives, plasma jet velocities around 10 km/s and electrical conductivities in the 10-kS/m range were anticipated.<sup>8</sup>

The simple experimental setup is shown in Fig. 11. Here, a shaped charge explosive is attached to a schedule-40 polyvinyl chloride (PVC) pipe that includes a time-of-flight circuit and an inductive probe. The overall length of these expendable units was 1.2 m, and end caps were attached such that a rough vacuum could be established in the pipe before a test. The time-of-flight circuit consisted of two wires traversing the pipe with a separation distance of 10.0 cm. The first wire was located 216 mm from the shaped charge exit plane. A dc voltage (24 V) was applied to these wires, and the current was measured across appropriately sized ballast resistors. When

the plasma jet broke each wire, the respective current signals would fall such that time of flight between the two locations could be measured. A time of flight of 10 ms is expected for a jet velocity of 10 km/s. Note that the search coil response function is the product of conductivity and jet velocity; therefore, even though the velocity of the calibration slug was approximately 0.002 km/s, the higher conductivity of the aluminum compensates, resulting in an appropriate calibration response function. Based on the output from the

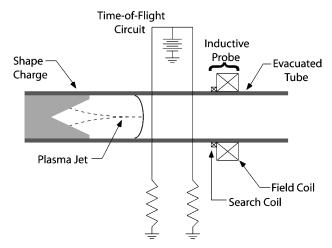


Fig. 11 Expendable explosive unit schematic for measuring plasma jet electrical conductivity.

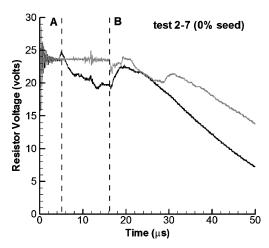


Fig. 12 Representative time-of-flight wavefrom for typical explosive plasma jet.

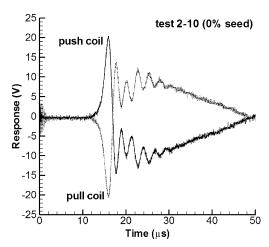


Fig. 13 Representative push-pull search coil waveforms for typical unseeded explosive plasma jets.

calibration and the high-explosive tests (discussed at the end of this section), the ratio of test data to calibration data is about 20:1 for unseeded and 100:1 for seeded shaped charges. Although this is not an ideal calibration, it is within reason for the precision required in this experiment.

Expendable coil forms were fabricated and mounted to the PVC pipe and field coils, and search coils were wound on these forms. The dimensions of these expendable probes were identical to that used in the calibration work. The probes were located 381 mm from the shaped charge exit plane. The tests were conducted in a blast chamber located at Accurate Arms Company test facilities in Bucksnort, Tennessee. To conduct a test, the assembled explosive

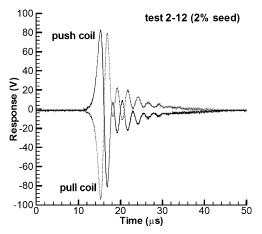


Fig. 14 Representative push-pull search coil waveforms for typical seeded (2% seed) explosive plasma jets.

units were placed in the blast chamber and a 200-ms delay initiator was attached to the end of the shaped charges. The delay initiator was necessary to ensure that the plasma jet passed through the probe during the desired target window at maximum field coil strength. It took slightly less than 200 ms for the solenoid-controlled relay to close. A rough vacuum would be established in the pipe, and a dual-circuit switch would be activated to excite the field coil and initiate the explosion. The data from the experiment were captured on a 4-channel digital oscilloscope.

Representative time-of-flight waveforms from a typical test are shown in Fig. 12. A denotes the first trip wire, and B denotes the second trip wire. In this particular test without seed, the time of flight was 11  $\mu$ s, yielding a jet velocity of 9.1 km/s, which is very near the anticipated value of 10 km/s. Of the 12 time-of-flight measurements taken, the 95% random uncertainty confidence interval within which the mean value will fall is  $8.9 \pm 0.7$  km/s. The average value of the four usable conductivity tests was  $9.4 \pm 0.8$  km/s, with 95% confidence. 19 Because the calibration velocity data were an order of magnitude lower than the test data, and due to physical constraints presented by the testing, the authors were unable to determine a valid way of estimating the systematic uncertainty in the experiment; therefore, the uncertainty values only include expressions of the random uncertainty in the experiment. However, because the goal of these experiments was to attain order-of-magnitude estimates for the magnetic Reynolds numbers, this type of uncertainty analysis is appropriate.<sup>19</sup> Note, however, that these mean values were not used in any calculations. The calculations for electrical conductivity use each test's own velocity measurement.

To avoid any possible electrostatic effects on the probe measurements, the push–pull search coil configuration was used for all tests. Representative waveforms from a typical test are shown in Fig. 13. In this test, there was no seed in the charge, and the entire response occurs within 30  $\mu$ s. The peak signal amplitude is about 22 V, and

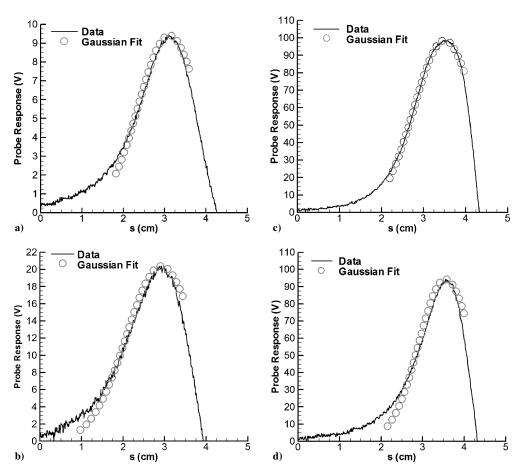


Fig. 15 Gaussian fit to entrance perturbation for a) test 2-4 with no seed, b) test 2-10 with no seed, c) test 2-6 with 2% potassium carbonate seed, and d) test 2-12 with 2% potassium carbonate seed.

Table 1 Summary of plasma jet measurements based on the peak-voltage method

Parameter	Test index number				
	2-4	2-10	2-6	2-12	
Seeding, %	0	0	2	2	
u, km/s	9.1	10.0	9.5	9.1	
$\sigma^*/\sigma_c$ , $10^{-5}$	7.8	14	76	78	
$\sigma^* u / \sigma_c u$ , $10^{-2}$	3.3	6.5	34	33	
$\sigma^*$ , kS/m	2.6	4.6	25	26	
$\mu\sigma^*u$ , m <sup>-1</sup>	29	58	300	295	

Table 2 Summary of plasma jet measurements based on the integral ratio method

Parameter	Test index number				
	2-4	2-10	2-6	2-12	
Seeding, %	0	0	2	2	
u, km/s	9.1	10.0	9.5	9.1	
$\sigma^*/\sigma_c$ , $10^{-5}$	5.9	11	58	56	
$\sigma^* u / \sigma_c u$ , $10^{-2}$	2.5	5.1	26	24	
$\sigma^*$ , kS/m	1.9	3.6	19	18	
$\mu\sigma^*u$ , m <sup>-1</sup>	22	46	228	210	

the push–pull signals are symmetric as expected. The signal oscillations appear to be indicative of tears in the plasma jet due to a strong axial gradient in velocity. This result could be anticipated from the work of previous researchers in the field. Waveforms obtained using seeded shaped charges were similar except for the fact that the peak signal amplitudes ranged from 90 to 100 V (Fig. 14). This indicated a substantial increase in electrical conductivity, as was expected.

In all cases, the entrance perturbation closely resembled a Gaussian response indicating an abrupt and very strong increase in ionization behind the shock front. Attempts to perform Gaussian fits on the entrance perturbations are shown in Fig. 15 for cases with and without seed. It is clear that the rising edge is more nearly Gaussian, whereas the falling edge is extremely abrupt. However, the Gaussian does tend to fit slightly beyond the peak amplitude in all cases.

The results of four tests are summarized in Tables 1 and 2 and are based on the peak-voltage and integral methods, as discussed in the "Mathematical Theory of Operation" section. Two of the tests are without seed (tests 2-4 and 2-10) and two tests are with seed (tests 2-6 and 2-12). The measured jet velocities are also shown. When it is assumed that the entrance perturbations can be reasonably approximated by a Gaussian, the peak electrical conductivity values in the plasma jet can be inferred using the peak-voltage method and Eq. (4), where it is implicitly assumed that  $\Psi_1$  is equal to unity. The results for the integral method are based on Eq. (5) with  $\Psi_2$  taken as unity. As stated in the "Mathematical Theory and Operation" section, the integral method is thought of as a more conservative calculation, based on previous work.  $^{11}$  The mean value for the calibration velocity was used for the purposes of Tables 1 and 2.

The peak electrical conductivity was found to be in the range of 4 kS/m for unseeded charges and in the range of 20 kS/m for seeded charges. When the measured jet velocity was used, it was estimated that the product  $\mu_0\sigma u$  can be as high as 60 m<sup>-1</sup> for unseeded charges and 290 m<sup>-1</sup> for seeded charges. This implies that significantly high values for the magnetic Reynolds number  $Re_m$  can be achieved in moderate-scale devices, that is, devices that are measured in fractions of a meter.

#### **Conclusions**

An electrodeless inductive probe for measuring plasma jet electrical conductivity was designed, fabricated, calibrated, and tested.

The 25.4-mm-diam probe uses an excitation solenoid coil to produce an axisymmetric field within the bore, and a search coil is located nearby to detect field perturbations when the plasma jet enters the active probing region. The response function for the probe was obtained through a calibration procedure in which a metal slug of known conductivity was propelled through the probe with a known velocity. The electrical conductivity of actual plasma jets were then determined using the calibration data and assumed conductivity distributions for which approximate analytical solutions are known

Exploratory laboratory experiments were conducted using seeded and unseeded shaped charge explosives. The jet velocity was inferred from time-of-flight measurements, and the electrical conductivity was determined from measurements using the inductive probe. The average jet velocity was about 9 km/s, and the measured conductivities were in the range of 3 that kS/m for unseeded charges and in the range of 20 kS/m for charges seeded by mass with 2% potassium carbonate. The peak electrical conductivity was found to be in the range of 4 kS/m for unseeded charges and in the range of 20 kS/m for seeded charges.

It is concluded that this inductive probing technique represents a very useful tool for investigating plasma jet ionization characteristics. Based on the results obtained in the exploratory experiments, it is further concluded that high-explosive plasma sources are capable of producing very high magnetic Reynolds number jets that can be of significant use in the development of magnetic flux compression reactors for pulsed propulsion and power.

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